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Physics and Mathematics Behind Tomographic Techniques of Medical Imaging

Thomas Reid

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By

Thomas Reid

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Chemistry, Mathematics, and Physics

X:_____

Dr. James Peters

X:_____

Dr. Sheldon Rothman

Date:_____

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Abstract

Aside from that of x-rays, the important methods of medical imaging provide 3-dimensional views of the human body. The mathematics behind this is called reconstructive tomography. Indeed, "C.A.T." is an anagram for computed axial tomography. There are two other important tomographic techniques in use. Magnetic resonance imaging and positron imaging. In this thesis we explain the physics and some of the mathematics behind each.

Introduction

There have been approximately 25 individuals who were awarded a Nobel prize for scientific discovery that directly or indirectly contributed to the advancement in medical imaging. A number of them are mentioned in this paper. The most important in regards to this paper would be the discoveries made by Allan M Cormack, Godfrey N Hounsfield, Paul C Lauterbur, and Peter Mansfield. Allan M Cormack and Godfrey N Hounsfield in 1979 were awarded the prize in medicine for their CT scanner. Paul C Lauterbur and Peter Mansfield in 2003 also earned the prize in medicine for MRI.

Some of the methods of imaging a persons or objects internal structures were originally used for some other purpose and found to be useful in this area as well. An example of this is lunar occultations. The term occultation is most frequently used to describe those relatively frequent occasions when the Moon passes in front of a star during the course of its orbital motion around the Earth. Early radio astronomers found occultations of radio sources by the Moon valuable for determining their exact positions, because the long wavelength of radio waves limited the resolution available through direct observation. Through occultation they could filter out the long wave frequencies in the background with the moon and receive wanted signals as

the moon allowed them to pass again. This was crucial for the unambiguous identification of a radio source and allows the determination of many things such as the occluded objects position, size, temperatures, etc.

Computed Tomography: C.T. Machines

A medical device of importance widely used in many hospitals is known as a C.T. scanners which stands for Computerized Tomography. Understanding the meaning of computerized tomography gives an accurate description of what these scans represent. Computerized just refers to the data collection and processing that is done by a computer while tomography refers to a technique for displaying a representation of a cross section through a human body or other solid object using x-rays or ultrasound. In other words, a C.T. scan uses x-rays that are shot linearly through the human body along multiple positions or angles. The analysis of the x-rays passing through each line is reconstructed into a 2-dimensional cross section by a computer with the data collected by the paths of the x-rays, This gives rise to the term tomographic reconstruction.

How the machine is able to do this is based primarily on its structure. A common description of C.T. machine nowadays would be a medically clean looking bed with a huge, donut shaped, tube at the head of the bed. The bed is on a rail system so that anyone or anything laying on it can be slid into the donut shaped tube. The operator of the C.T. machine usually controls it from behind panels of glass, some distance away by means of a computer. While a scan is taking place the subject must remain perfectly still while a series of loud and strange noises occur. These noises are the machine in action. Inside the donut shaped tube are countless mechanical and electrical components, the most important aspects of this cluster is the

understanding that there are a series of "sweeping" x-ray guns and directly opposite them are detectors with the subject directly in the middle. While the machine fires x-rays through you and picks up the transmitted rays it is moving to different positions or angle and repeating the process in order to have a full 360 degree image of the subject for that cross-section. To accomplish this the ray guns and detectors only move a full 180- degrees with very thin increment in between positions.

Godfrey Hounsfield joined the Electric and Musical Industries (EMI) company, where he worked on radar and computers until 1967. In 1967 Godfrey's previous projects ceased to be of interest to EMI because of changes in company strategy. He was asked to suggest a new line of work involving pattern recognition, and he suggested what eventually became CT scanning. Hounsfield proposed that by using x-rays one could take multiple exposures around an object to determine its internal structure. EMI was unenthusiastic because they had no significant medical business, Hounsfield had no medical knowledge, and his proposal was a high-risk leap beyond existing technology. So they sought external funding, and Hounsfield managed to get a small amount of funding from the UK Government Department of Health and Social Security. His struggles for financing continued for the next four years, and he also had to struggle against apathy from most of the medical profession. Visits to radiologists at many leading hospitals found that almost everyone (with the notable exceptions of James Ambrose, Louis Kreel, Evan Lennon and Frank Doyle) thought that his proposal was pointless.

James Ambrose, a neuroradiologist at Atkinson Morley Hospital, in London, would go on to assist Hounsfield in the various prototypes of the CT scanner. Initially, the original prototype CT scanner was used to examine brains obtained from a histopathology museum. However,

preserved brains were not the ideal candidates for mimicking the living human brain. Bovine brains were sought as a solution to this problem. Unfortunately, the electric shock given to the cows in the abattoir resulted in diffuse cerebral hemorrhage, again not a good model for the living human brain. Finally, the idea to use ‘kosher’ slaughtered cows came into being, with cows killed under strict kosher guidelines not suffering widespread hemorrhage, resulting in the first promising scans of the brain in 1968.

In 1968 Hounsfield was granted UK Patent No. 1283915 for "A Method of and Apparatus for Examination of a Body by Radiation such as X or Gamma Radiation" for his new scanner and its technology.

The first clinical CT scan was conducted on 1st October 1971 at the Atkinson Morley Hospital, demonstrating a well-defined cyst.

Everything changed after he presented his initial CT scans at the annual congress of the British Institute of Radiology conferences in London on April 1972 and New York in May 1972. As soon as people saw these images, they realized the groundbreaking potential of this new technique. He was the first to show discrimination between soft tissues, tumors and blood clots in clinical use at acceptable cost and dose. The concept presented by Hounsfield involved a thin cross section of the head, a tomographic slice, which examined from multiple angles with a pencil like x-ray beam. The transmitted radiation was computed by a scintillation detector, fed into a computer for analysis by a mathematical algorithm, and reconstructed as a tomographic image.

Hounsfield was awarded the Nobel Prize in Physiology or Medicine in 1979. The Nobel was co-awarded to Allan M Cormack, a South African physicist who, unbeknownst to Hounsfield

had developed the theoretical mathematics underpinning the CT scanner. Cormack published his important paper 'Representation of a function by its line integrals, with some radiological applications' in 1963 in the Journal of Applied Physics.

His interest in the theoretical aspects of x-ray technology while working at the Groote Schuur Hospital in Cape Town led to important theoretical publications in this new area and the subsequent sharing of the 1979 Nobel Prize in Physiology or Medicine, with Godfrey Hounsfield, for the development of CT.

The technique presented by Hounsfield has changed very little but has become more efficient and practical in modern applications. The C.T. machine as a whole is a remarkable imaging device that allowed new examination of the human body never before seen in an x-ray image. It demonstrated a radiographic difference in the various soft tissues of the human body such as blood clots, grey matter, white matter, cerebrospinal fluid, tumors, and cerebral edema all appeared as separate entities. The soft tissues could no longer be assigned the physical characteristics of water.

There have been about four-generations of C.T. machines since they were proposed. Each generation follows the same idea and technique but have adapted to a newer process of taking scans in order to become more efficient and reliable. Each generation can be discussed in terms of their scanning motions. These scanning motions are translate-rotate with one detector, first generation. Translate-rotate with multiple detectors, second generation. Rotate-rotate, third generation and rotate-fixed, the fourth generation. The first generation was the original EMI unit. It employed a pencil-like x-ray beam and a single detector; one detector per tomographic section. The x-ray tube detector movements were both linear and rotary. A five-view study of the head

took about 25-30 minutes. The major difference from this generation and all further generations was to shorten the scanning time for each tomographic section. This was done by using a fan-shaped beam with multiple detectors instead of a pencil-like beam and a single detector. The number of detectors and fan-shaped beams varied on manufacture at the time but the numbers translated to how quickly a scan can be done. The first generation using one pencil beam and detector had to move about three to five degrees between each scan up until 180 degrees was achieved. If a second generation was used with 30 fan shaped beams and detectors then it could move 30 degrees between each scan and achieve a full tomographic scan within 6 scans instead of 180 scans. The second generation still moved in a similar fashion as the first generation in a translate-rotate motion. The third generation switched to a rotate-rotate motion. This involved multiple fan beams and detectors that both rotate around a subject at the center of the rotation. The original third generation used 288 detectors and a fan like x-ray beam and could produce a scan in 4.9 seconds. Newer ones use over 700 detectors. This style allows a single image to be computed from many projections which was often more than a thousand projections. The fourth generation is a rotate-fixed movement which forms a 360 degree ring of detectors and a fan like beam moving around the patient. The x-ray beam is continuously on for the duration of the scan and can read greater than a thousand projections in a second. Even though these machines are listed as generations there are no clear advantage from a third generation and fourth generation. They are still commonly being used and having their own advantages over the other. The main improvement from the different generations was the use of the fan like x-ray beam and multiple detectors instead of just one which undoubtedly increased the speed of the scanning but it does

lead to an increase in the amount of scattered radiation. Using fan beam scanners are more likely to record the scattering and lead to artifacts or noise in the image.

Method of Proofs

The major part of a C.T. scan is the fact that it is computerized. There is a computer that is fed all the data that is analyzed in an algorithm to reconstruct an image from x-ray beams. There is no need to go in depth with these formulas or understanding of the mathematics but a general understanding will suffice for this paper. For both medical imaging and radiotherapy, it is standard to view the body as a series of two-dimensional cross sections called slices. This means for each slice there is some density f(x,y). In medical imaging the function f(x,y) is unknown and we want to determine it, to the highest possible degree, with the use of line integrals. The line integral data corresponds to x-ray or γ-ray data for CT and PET respectively. For MRI it is the radio wave along a given line. In all cases, it is a photon beam. For radiotherapy, we know what f(x,y) is and we want to bombard a tumor with minimal radiation to surrounding healthy tissue.

For Lines Parallel to the x-axis we have

$$L(y)= \int f(x,y)\, dx$$

While for lines parallel to the y-axis this becomes

\ $$L(x)=\int f(x,y)\, dy$$

For other directions, we need to parameterize the lines. Let θ be the angle as measured from the positive x-axis, and let $U_\theta = \cos(\theta)\hat{i} + \sin(\theta)\hat{j}$ be the unit vector in the direction θ. Given a point (x,y) with a position vector V= xi + yj we define $L(\theta, t) = \{(x,y) : V \cdot U_\theta = t\}$. For the point (a,b)

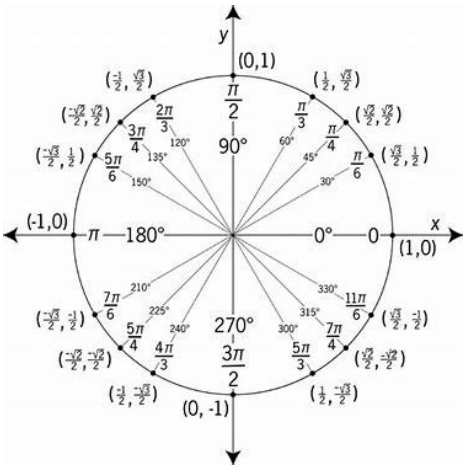
parallel to U_θ we have $t = \sqrt{a^2 + b^2}$. Every other point (x. y) in $L(\theta, t)$ is on the line with perpendicular vector U_θ at a distance t from the origin.

The Radon or X-ray transform of f(x,y) is obtained by integrating f(x,y) over the line $L(\theta,t)$. We write

$$L(\theta,t) = \int f(x,y)\omega(\theta,t)$$

where the differential form $\omega(\theta,t) = \cos\theta\, dx + \sin\theta\, dy$.

Note $\theta = 0$ gives dx; $\theta = \pi/2$ gives dy.



For polar coordinates, we let θ vary from 0 to 2π. For tomography, symmetry suggests we restrict to angles between 0 and π, since going from π to 2π would only be redundant in our measurements and calculations, with t representing signed distance from the origin. Strictly speaking, the data obtained in tomography is a set $L(\theta_k, t_m)$ which varies over finitely many values of k and m. For the density functions encountered in the body, the transform data obtained has the property that for each fixed θ, $L(\theta,t)$ is continuous in t except at a finite number of points. As such, it is the convention to think of $L(\theta_k, t)$ being known for all t in a given direction, this determines a profile.

The other part of computing is eliminating errors and understanding what the photon beams being used do. A photon beam passes through or strikes a medium, traveling a total distance D through the medium. We write $D = n\,\Delta s$ where n is very large and each increment of Δs is very thin. The probability of a photon being absorbed or scattered in each increment is p,

the probability the photon gets through is $(1-p)^n$. For when Δs is sufficiently small, we have p close to zero.

The approximation

$$(e^{-p}) \approx 1-p$$

Gives the probability of transmission

$$(e^{-p})^n = e^{-pn} = e^{-\delta T}$$

Where δ is called the coefficient of attenuation for the material. The value of δ is proportional to the imaginary component of the refractive index.

So by knowing the refractive index of materials we can predict the probability of the photons being transmitted as well as reflected. Suppose it can be shown that the probability that a photon is reflected is

$$p = \frac{(n-1)^2}{(n+1)^2}$$

If we use the example of glass, with a index of refraction n = 1.5 and two surfaces we can determine the probability of reflection.

$p = \frac{(1.5-1)^2}{(1.5+1)^2} = 0.04$ for one surface, double it to determine the probability of the sheet of glass so see that there is approximately a 0.08 probability of the photons being reflected and a 0.92 probability of the light being transmitted. This makes sense since we expect light to come in through our windows and not be reflected or absorbed by an object that has a higher index of refraction. This logic is being applied to the photon beams passing through the patient's body.

The actual data obtained is beam attenuation which is the ratio output (I_{0t}) to input (I). photon density of x-rays. According to Beer's Law I_{0t}/I is proportional to expL(θ,t). To prove this we'll start with a form of Beer's Law which states $I=I_0e^{-\mu x}$ where I is the detected x-ray

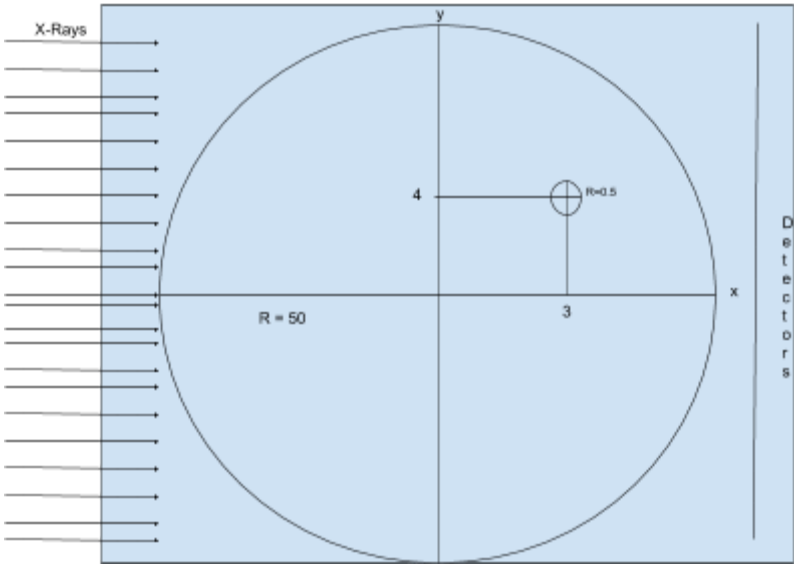
intensity, I_0 is the starting x-ray intensity, μ is the materials linear attenuation coefficient and x is the length of the x-ray.

This equation can be rearranged to $\mu = \ln(I_0/I)$. If we eliminate the natural log by implementing exp we get $e^{\mu}=I_0/I$ which is why I_{0t}/I is proportional to expL(θ,t).

Hounsfield values are proportional to beam attenuation Thus, H=900 is double the x-ray density of H=30. Some Hounsfield values are shown in the chart below

Material	**Hounsfield Unit**
Air	-1000
Lung	-500 to -200
Fat	-200 to -50
Water	0
Blood	25
Muscle	25 to 40
Bone	200 to 1000

Suppose we have a small region, a tumor perhaps of slightly higher density than the surrounding tissue. Lets say a density of 1.0 versus 1.25 as shown below

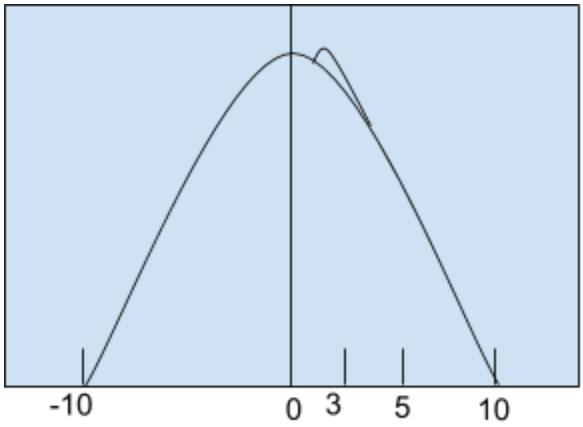


So f(x,y) = g(x,y) + h(x,y)

where g(x,y) = 1 for $x^2 + y^2 \leq 100$ (in cm) and that h(x,y) = 0.25 for $(x-3)^2 + (y-4)^2 \leq 1$.

For the indicated direction θ=1i + 0j we have $L(\theta,t)= 2\sqrt{100 - t^2}$ for $-10 \leq t \leq 3$ and

$5 \leq t \leq 10$. For t to be between 3 and 5, $L(\theta,t)= 2\sqrt{100 - t^2} + 0.5\sqrt{1 - (t-4)^2}$



Note that for θ= 4/5i-3/5j; we get

$$L(\theta,t)= 2\sqrt{100 - t^2} + 0.5\sqrt{1 - t^2}$$

for $|t| \leq 1$ which makes the region easier to detect. By Beer's Law the change in attenuation is proportional to exp(0.5) for t=0 which is easier to detect than a difference of $L(\theta,t)= 20$ and $L(\theta,t)= 20.5$

For each fixed θ

$$\int L(\theta, t)\, dt = \iint x\, f(x,y)\, dy\, dx = D$$

The total density of the slice. A change of θ should also give the same value of D so we have a simple way to check for errors or otherwise known as "noise."

For $\theta = 0$ we have

$$\int t\, 1(0, t)\, dt \;=\; \iint x\, f(x, y)\, dx\, dy \;= \mu_y D$$

where μ_y is the y component of the centroid. If the patient moves the (μ_x, μ_y) will change as a consequence and allows the algorithm self correct for the change in patient.

The task of reconstructive tomography is to recover a good approximation f(x,y) to form sufficiently many profiles. Another way to think of this without the math is to imagine taking a photo of an object in front of you, but not all the photos are good. You have some that are blurred. If you were to superimpose all of the images you took onto one another there would be a more distinguished image of the object since the superimposed images will form a contrast between the object and the surroundings. In a way it will bolden the image instead of having a blurred image. In doing so we can get a better representation of the internal structures of a patient with a good approximation even with slight movements from the patient not being able to remain absolutely still.

Back Projections

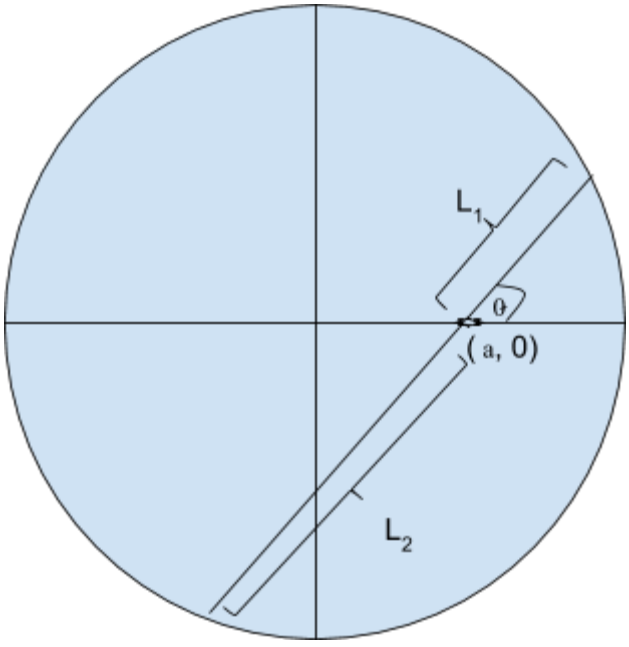
While the method of direction profiles seems like a good way to detect boundaries of regions with differing densities, the most common method of tomographic reconstruction is based on back projection

Given a density f(x,y) in the plane, for each point (α, β) we find the line integral passing through the point at a direction θ from the positive x-axis and then integrate with respect to theta for $0 \leq \theta < \pi$. This determines a function $F(\alpha, \beta)$ at each point in the plane. Note that this included points where f(x,y) = 0 (outside the object). The fact that $F(\alpha, \beta)$ is not 0 at such points while f(x,y) is, leads to an artifact called streaking.

Suppose f(x,y) = 1 for $x^2 + y^2 \leq 1$ and 0 for $x^2 + y^2 > 1$. Since the region is spherically symmetrical, we take $\beta = 0$ and α on the positive x-axis. Fro $\alpha = 0$, we get lines of length 2 through the origin. This gives

$F(0) = \int 2 d\theta = 2\pi$.

For $\alpha < 1$ we need to find the length of the line segment L_1 above the x-axis and that of the line L_2 below the x-axis (see diagram) and compute the sum.



By the Law of Cosines

$\alpha^2 + L_2^2 = 1^2 + 2\alpha L_2 \cos\theta$. The quadratic formula gives two roots. The larger root is $L_2 = \alpha\cos\theta + \sqrt{\alpha^2 cos^2\theta - \alpha^2 + 1} = \alpha cos\theta + \sqrt{1 - \alpha^2 sin^2\theta}$. For the region above the x-axis, we replace $\cos\theta$ by $\cos(\pi - \theta) = -\cos\theta$ to get

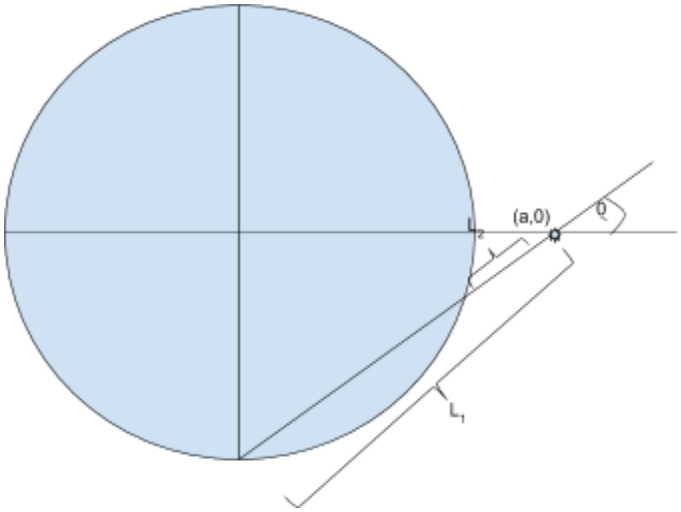
$L_1 = -\alpha cos\theta + \sqrt{1 - \alpha^2 sin^2\theta}$.

The total length is $2\sqrt{1 - \alpha^2 sin^2\theta}$ so $F(\alpha) = 2\int \sqrt{1 - \alpha^2 sin^2\theta}\, d\theta$. For $\theta = 0$ we get

$F(\alpha) = 2\sqrt{1 - \alpha^2}$ which is the same as the line integral obtained by direction profiles.

Typically, this elliptic integral is evaluated by using numerical methods. The integrand is a decreasing function of α for each fixed θ and F(1) equals two times the integral for $\cos^2\theta$ for 0 to π so $F(1)=\pi$. The integral for θ between $\pi/2$ and π reverses the roles of L_1 and L_2 so we can double the integral and integrate from 0 to $\pi/2$.

For $\alpha > 1$, the line integral is the length of the line L_1 ,imus that of L_2 as shown below



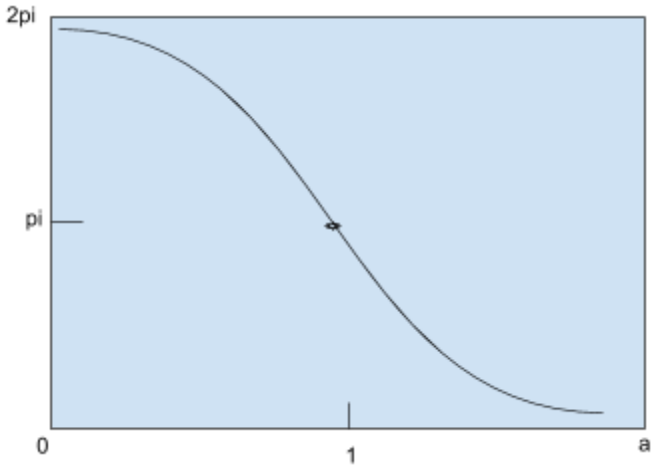
Here we have the quadratic $\alpha^2 + L^2 = 1^2 + 2\alpha L\cos\theta$, where the larger root is L_1 and the smaller root is L_2. The line does not intersect the region for $\alpha^2\sin^2\theta > 1$. Accordingly, the limits of integration are from 0 to $\arcsin 1/\alpha$. We get

$$F(\alpha) = 4\int \sqrt{1 - \alpha^2 sin^2\theta}\, d\theta$$

for $\alpha > 2$ we can make the approximation $\sin\theta = \theta$. This gives an integrand $\sqrt{1 - (\alpha\theta)^2}$ integrating from $\theta = 0$ to $1/\alpha$, and putting $u = \alpha\theta$ we get

$$F(\alpha) \approx 4\int \sqrt{1 - (\alpha\theta)^2}\, d\theta = 4/\alpha \int \sqrt{1 - u^2}\, du = \pi/\alpha$$

by differentiating under the integral we see that $F'(\alpha)$ approaches $-\infty$ as α gets close to 1. Thus, $F(\alpha)$ has a vertical inflection at $\alpha = 1$. Since $F(\alpha)$ is positive and decreasing we get the graph below.



As might be expected, it makes sense to differentiate F(α) to detect boundaries. Differentiating F and then taking its Hilbert transform yields filtered back projection data, which well approximates f(x,y). In this case, f(x,y) = f(r) where $r^2=x^2+y^2$. The HIlbert transform of a function $\phi(x)$ is given by

$$H_\phi(x)=\int \phi(y)/(x-y)\, dy$$

where the limits of integration are -∞ to ∞. We show that the Hilbert transform reverses parity. We have

$$H_\phi(-x)=-\int \phi(x)/(x+y)\, dy$$

If we let u=-y, this becomes

$$H_\phi(-x)=-\int \phi(-u)/(x-u)\, (-du)$$

with the order of integration reversed. Equivalently,

$$H_\phi(-x)=-\int \phi(-u)/(x-u)\, du$$

integrated from -∞ to ∞. But if ϕ is an even function, we can replace $\phi(-u)$ by $\phi(u)$ to get $H_\phi(-x)=-H_\phi(x)$. In the same way, it follows that if f is odd then $H_\phi(-x)=H_\phi(x)$

In our example, F'(α) is odd so its Hilbert transform has the same parity as f(α). It can be shown that up to a constant multiple, the Hilbert transform of F'(α) is f(α).

Scatter and Absorption

There are many things that can occur while undergoing a C.T. scan. The most important things you want to accomplish from a C.T. scan are a quick and accurate scan that shows as

much detail of the subject as possible. We've already talked about how the scanning process has been made quicker however the use of those designs can lead to an increase in scatter radiation. There are a few types of scatter radiation that occurs when sending x-rays through a solid object. Some of them are compton scattering, dissipative absorption or the photoelectric effect, photomultiplier effect, and non-resonant scattering or Rayleigh scatter. Each type of scatter radiation occurs from the different interactions of the x-ray beam and the solid object it is passing through.

Compton scatter is one of most concern when it comes to the different forms of scatter. This is because it can create noise on the scans. Compton Scatter is similar to the elastic collision between billiard balls in the game of pool. This scattering occurs when an X-ray hits an electron, causing the release of a photon from the collision as well as the initial X-ray being redirected at a new angle. This new angle is classified as a front scatter, side scatter, or a backward scatter depending on the direction it becomes redirected. The problem with this form of scatter is that it turns one ray being shot in a straight line into a recording of two rays originating from the point of collision.

The photoelectric effect is when a medium absorbs a photon. This absorption occurs at an energy level that causes electron shell transition, when an electron becomes excited and jumps to another orbital. Typically, this will supply the electron with kinetic energy that will be lost within the object as heat, however, if the electron is hit with enough intensity or energy it could cause the electron to jump to a higher energy state. This will make the electron unstable and it will jump back to its normal energy state and release a photon that is at a lower intensity than it was before the interaction with the electron. If the electron is raised several energy levels then it

can jump back to its original energy state in different ways. Each step transition to a lower energy level will release a photon. If these lower intensity photons are able to escape the object and not dissipate as heat are considered to be the photomultiplier effect.

Rayleigh scatter is another form of electromagnetic scattering. It is when a low energy photon excites a molecule causing it to emit another photon of equal energy. This elastic scatter is in all directions for thin gases but mainly occurs in the forward or reverse reflected at boundaries of denser materials. This is likely to occur since Rayleigh scattering favors low energy and short wavelength photons and X-rays are high energy with short wavelengths. This form of scatter is best demonstrated by the blue sky. The color blue is of a shorter wavelength than visible light and so is easily scattered throughout the sky that contains thin gases.

Choosing the right energy levels and exposure time for the x-ray beam is like a balancing act such that you need to choose what is a more desirable image during the time of scanning. The major importance is the amount of radiation exposure the subject receives, the radiation scatter effects, spatial resolution, and contrast resolution.

P.E.T.

A P.E.T. scan or Positron Emission Tomography scan is a style of C.T. scan that is widely used for diagnostics. P.E.T. builds images by detecting energy given off by decaying radioactive isotopes.

Isotopes are atoms of an element with the same number of protons (positively charged particles) in the nucleus but a different number of neutrons (neutral particles). Because radioactive isotopes are unstable, as they decay they throw off positrons that collide with

electrons and produce gamma rays that shoot off in nearly opposite directions through a process called pair annihilation.

PET systems use the paths of the two detected gamma rays to determine the originating collision point, a process called electronic collimation. The scanners use a circular series of gamma ray detectors to envelop the patient to detect both gammas so the instrument can use electronic collimation to predict where the energy signal originated. This signal is then converted into a three-dimensional image slice.

This is made possible with some assumptions and the laws of conservation of momentum and energy.

$$P = MV$$

where P is momentum, M is mass and V is the velocity of a given object.

To use this law with two objects colliding it is adapted to

$$P_{total} = M_e V_e + M_p V_p$$

were P_{total} is the total momentum and the masses and velocity are respectively assigned.

As a system, the pair has an initial kinetic energy of:

$$K_0 = ½\ mV_e^2 + ½\ mV_p^2$$

After annihilation the energy is increased by $2mc^2$ and the momentum is conserved. However, K_0 is now assumed negligible because $2mc^2$ is of the order $(10)^{-13}$ Joules while V_p and V_e have magnitudes of order $(10)^5$ or $(10)^6$ m/s. This means that K_0 would be of the order less than $(10)^{-18}$ Joules, an extremely small magnitude and so can be ignored.

Assuming the annihilation process produces exactly two photons, which almost always occurs in practice, the final energy is:

$$h\frac{c}{\lambda_1} + h\frac{c}{\lambda_2} = 2mc^2$$

This suggests that the wavelengths λ_1 and λ_2 are of the order $(10)^{-12}$ m suggesting the release of gamma radiation.

The initial momentum of this system is also assumed negligible compared to $|h/\lambda_1|$ and $h/\lambda_2|$ because

$hc/\lambda = mc^2$ yields $h/\lambda = mc$

which is far greater than |mV|

Since the initial momentum is essential zero, the pair must fly apart so as to maintain a zero total vector momentum and preserve the law of conservation of momentum. This requires that the photons travel in opposite directions at an angle of 180° apart and have equal magnitudes of momentum. This means that

$|h/\lambda_1| = |h/\lambda_2|$ so $\lambda_1 = \lambda_2$

Using this in the equation below that we acquired above:

$h\frac{c}{\lambda_1} + h\frac{c}{\lambda_2} = 2mc^2$ simplifies to $\lambda = hc/mc^2 = h/mc$

This simplified equation gives the Compton wavelength and is what allows for the practical use of P.E.T. scans.

The applications of P.E.T. scans are those of diagnostic techniques. To perform a P.E.T. scan, a patient is injected with radioactive isotopes that make their way into the bloodstream. The isotopes are unstable and so emit or release positrons as they decay. These positrons come into contact with electrons, their particle-antiparticle counterpart, and emit gamma rays for detection. What allows this to be useful is the rate at which different parts of a living organism uptakes nutrients from its environment. As a human being, our blood provides our living cells with

nutrients needed for survival. Different parts of our bodies require more or less of these nutrients in order to function properly and survive. Since the radioactive isotopes are infused with the blood, higher concentrations of them will be taken up by parts of the body that require more nutrients. The higher concentration of radioactive isotopes means there will be more positron emission in those parts and in turn more pair annihilation and release of gamma rays. This allows for the location of high nutrient dependent structure in the patient's body. This practice is most commonly used in detection and location of cancer or tumor cells since they require more nutrients from the body. Once a location is determined, a more depictive imaging technique is used to get better visual of the size and type of structure indicated by the P.E.T. scan, most likely a C.T. scan or an M.R.I. scan.

M.R.I

Mansfield in 1977 saw the first cross-sectional MRI image of a student's finger using a small scale machine. This opened doors to funding, allowing them to build a full-size machine. Mansfield had his abdomen scanned using line-scan MRI. Interestingly, no one was brave enough to have a scan due to fear of the magnetic field inducing a myocardial infarction.
A significant flaw in Mansfield's eyes was the speed with which an MRI produces images, and in 1977 he invented a technique where fast switching magnetic field gradients could be used to form an image significantly faster, known as echo-planar imaging. The first clinical scanner was built in 1980 and made available for clinical use in 1984. In 1963 Lauterbur joined the State University of New York (SUNY), in Stony Brook, with tenure in the Departments of Chemistry and Radiology, becoming a full professor in 1984. His main research focus was NMR spectroscopy, analyzing the composition of molecular structures, solids and liquids.

In 1971 after reading an article by Raymond V Damadian, Lauterbur became interested in the potential biological usages of NMR. Researchers had already shown that NMR could be used to distinguish malignant from non-diseased tissues or to assess blood flow. Until this time scientists had employed a uniform magnetic field, but Lauterbur realized that using a non-uniform field would allow the precise localization of structures within a sample to be determined. One of the first NMR images he took was of a clam.

In 1973 he successfully published his seminal paper on MRI in Nature (an earlier draft was rejected). In this paper Lautebur coined the term "zeugmatography" for MRI. As he explained the word was derived from the Ancient Greek ζευγμα (zeugma) 'that which joins together'.

MRI's are an interesting piece of technology that can be hard to understand since it incorporates abstract ideas and techniques in order to function the way it does. To try and explain, this paper will try to only touch upon the key aspects that are most important in understanding how it works. To begin, the patient is going to be considered positioned in a uniform magnetic field B directed perpendicular to the body cross sections. Taking these cross sections to be parallel to the x-y plane, the magnetic field has direction k or -k. The field must be sufficiently strong to cause many of the hydrogen nuclei (protons) to align in its direction. Typically, this magnetic field has a magnitude 0.5 to 2.5 T. Due to quantum mechanical constraints on the possible angular momentum states, the protons cannot align exactly the same direction. They precess (wobble) about what we visualize to be a 30 degree angle, returning to random orientation once the field is removed. The frequency of precession called the Lamour frequency, is given by

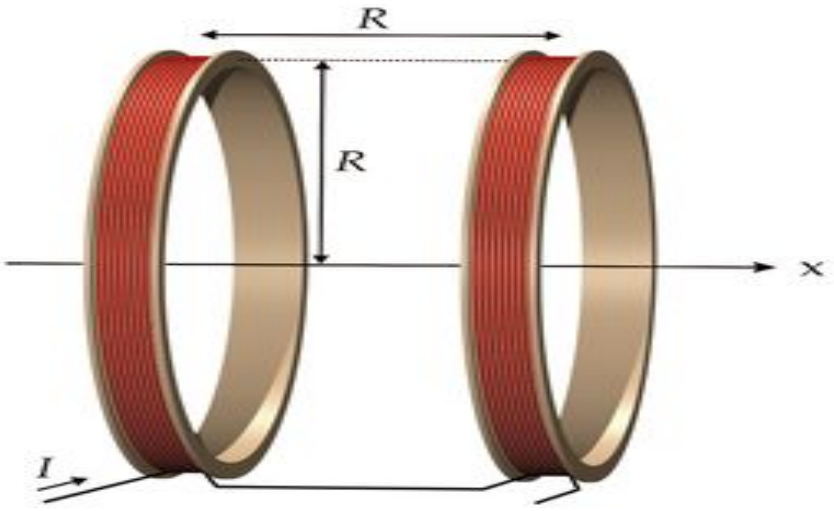
$$F=\gamma|B|/2R$$

where ɣ is the gyromagnetic ratio equal to $2.68(10)^8$ Hz/T for the hydrogen ion. To attain a field that is nearly constant over the length of the body requires a Helmholtz coil. Wrapping N turns of wire radially in a circular loop of radius r induces a magnetic field perpendicular to the loop having maximum magnitude

$B = \mu_0 NI/2R$

where I is the current, in amperes, in the wire and $\mu_0 = 4\pi(10)^{-7}$ Tm/A. This field has the desired direction but decreases rapidly with distance. Adding another circular loop at a distance R from the first, and having the same values of N and I, yields a Helmholtz coil. It can be shown that the superposition of the two fields is nearly constant between the coils.

Since the patient must lie between the coils, a typical value of R is 2 or more meters. Since $|B| \leq \mu_0 NI/2R$, the values of I and N must be very large to obtain Tesla level fields. We want $N \geq (10)^4$ and $I \geq (10)^3$. Since electrical resistance would require prohibitively high voltages, the circuit is supercooled using liquid nitrogen.



When the field is switched off, the protons emit photons at the Lamour frequency; this is the signal. For a one T field, we get 268 MHz, which fall in between the range of FM and cell

phone frequencies. The energy of each photon is given by E=hf where h= $6.626(10)^{-34}$ Js, this is Planck's constant. In units of electron volts this is of order $(10)^{-6}$ eV which is quite safe for the patient. By comparison, UV radiation is 3 to 5 eV; for CT scan the photons are more than $(10)^{5}$ eV.

Of course, if all the protons were emitting photons of the same frequency we could not determine different tissue. For this reason there are ordinary electromagnets called gradient coils that increase or decrease the field by 10 or 20 Gauss (1 Gauss = $(10)^{-4}$ T). Gradient coils in the z-direction allow us to identify x-y sections. Another gradient coil, say in the x-direction, provides a single frequency signal over the lines in the direction of the y-axis. Rotating the latter gradient coil gives lines in different directions. These determine the line integrals used for tomographic reconstruction of each f(x,y) section.

In the simplest case, we obtain what is called a proton spin density picture. We can also measure relaxation time for the ions, taking advantage of the fact that is will be different for hydrogen in different compounds.

Similarities and Differences

These three imaging techniques offer an enormous resource in understanding anatomical structure inside people. It provides physicians with the ability to observe and understand abnormalities anywhere in the body with a great ability to differentiate what kind of tissues are being imaged. The trick for the physician is to understand which imaging technique would be best for treating the patient and limit the time and resources wasted on unneeded practices. This means the physician must have an understanding of what each imaging technique provides, how

it can be useful in their treatment, as well as placing an importance or value on the results the images may provide.

We have already discussed to some length what each imaging technique provides us with; to summarize, a Positron Emission Tomography scan is ideal for locating positions where high electron density is and is useful as a broad surveying technique used mainly to locate forms of tumors or cancers that may be in the body. The other two techniques in a general sense have similar results that can be achieved through different methods. C.T. and M.R.I. allow detailed images of structures including soft tissue, however, there are advantages and disadvantages in choosing which method to use.

Understanding that C.T. uses radiation in the forms of X-rays and M.R.I. uses powerful magnetic fields and radiofrequency pulses to produce detailed pictures of organs and other internal body structures are the primary difference between the two methods. When choosing which method is better suited for the task at hand you must place value on what is most important in terms of time vs quality, a patient's needs, and resources at hand. In the situation of an emergency a C.T. scan might be the better suited method because it is many times quicker than if you were to use a M.R.I. while giving an image that will be useful to recognize internal complications. However, if you have enough time an M.R.I. might provide a more detailed picture of the internal structures of more soft tissues, internal organs, brain, etc. Some people aren't able to receive one or the other imaging methods. Some reasons for this can be a woman who is pregnant and should not expose her unborn child to forms of radiation so she can not receive a C.T. scan. Another complication could be a person with some kind of magnetic material who can not be placed in an M.R.I. machine otherwise the material could be ripped

from the person and damage the M.R.I. machine because of its very powerful magnetic field that it generates leading to many more complications. Then there is the problem of resources. M.R.I. machines are expensive devices and are expensive to use. There may not be many available at a given institution leading to a waitlist to use the machine and a person might not be able to afford the procedure of getting an image. This expense and availability weighs heavily on some decisions regarding the different imaging techniques.

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